

Study on an Efficient Template Placement Algorithm for CBC searches in GW data

Takashi Kato, Hideyuki Tagoshi

ICRR, University of Tokyo

Abstract

For searches of gravitational waves from compact binary coalescence, usually matched filtering with template bank is used. For faster and more accurate searches, we introduce a new algorithm for template bank construction based on a previous study^[1] using the geometry of parameter space. In our work, we use a technique for matrix computation and consider the distribution of template points. Finally, we confirmed that our new method is more accurate and computationally fast. This algorithm can also respond to instrumental updates rapidly.

Introduction - Template bank

In matched filtering method in GW searches, we evaluate

$$(s|h) = 4\text{Re} \int_{f_{\min}}^{f_{\max}} df \frac{\tilde{s}(f)\tilde{h}^*(f;\boldsymbol{\lambda})}{S_n(f)} \quad (\boldsymbol{\lambda} = (m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, \dots)) \quad (1)$$

Because we don't know the true parameter value a priori, we need to prepare theoretical waveforms for various parameter values - called "template bank".

In order to place template points properly, we usually evaluate "match" between two waveforms ($h_1|h_2$). Given a threshold for match (called minimal match (MM)), we compute template points to satisfy the threshold.

Background - Geometry of parameter space

Some studies suggest placement algorithms to use the geometry of parameter space introducing coordinates for $\boldsymbol{\lambda}$ - called geometric placement.

Such an algorithm becomes simpler by introducing new coordinates for parameters^[2]. The phase function $\Psi(f; \boldsymbol{\lambda})$ of a waveform would be written as linear combination of orthonormal bases $\{\psi_\alpha(f)\}$ and coefficients $\{c^\alpha\}$,

$$\Psi(f; \boldsymbol{\lambda}) = \sum_{\alpha} c^\alpha(\boldsymbol{\lambda})\psi_\alpha(f) \quad (2)$$

Because the coordinate for $\{c^\alpha\}$ becomes Euclidean from orthonormality of bases, the computation of template points on the new coordinate is simple. Previous studies show the efficiency of this method^[3].

Even in the case that we don't know the analytic formula of $\Psi(f; \boldsymbol{\lambda})$, we can identify $\{c^\alpha\}$ and $\{\psi_\alpha\}$ using singular value decomposition (SVD) as follows^[1].

1. Generate a lot of random samples for parameters, $\{\boldsymbol{\lambda}_i\}$.
 2. Compute discrete phase matrix, $\Psi_{ij} = \Psi(f_j; \boldsymbol{\lambda}_i)$.
 3. Execute SVD for the phase matrix. \implies search of orthonormal coordinates and bases
- Then we can construct template bank on the new Euclid coordinate for $\{c^\alpha\}$.

Improvement - Less computation and More accuracy

1. Fast computation of SVD

In previous method, we needed to execute SVD for a huge size matrix. It dominates computational time in template construction.

To reduce computational cost in SVD, we introduced an approximation technique for low-rank matrix which is proposed in computer science originally^[4]. In this method, we approximate a matrix $A \in \mathbb{R}^{M \times N}$ by $A' \in \mathbb{R}^{R \times N}$ ($R \ll M$) extracting principal components from original matrix.

Performance test

We tested this method for the matrix Ψ_{ij} which has 10000×16384 components. In my laptop, computational time of SVD was

	computational time of SVD
w/o approximation	10 hr.
w/ approximation	6 min.

Table 1: Performance of matrix approximation

with relative error $< 10^{-10}$. Approximate SVD needed only $\mathcal{O}(10^2)$ principal components in the original matrix. We confirmed that matrix approximation works effectively to reduce computational cost in template construction.

2. Flatten the distribution of parameter points

In previous method, we needed to use a lot of random samples, and the accuracy of template bank is affected by the distribution of random samples.

To improve accuracy, we tried to flatten the distribution of random samples in the parameter space NOT for $\boldsymbol{\lambda}$ BUT for \boldsymbol{c} . We tested three types of distribution of random samples for the mass and spin of binary objects,

1. Uniform distribution for $(m_1, m_2, \chi_1, \chi_2)$
 2. Uniform distribution for $(\tau_0, \tau_3, \chi_1, \chi_2)$ \cdots new parametrization for mass
 3. Uniform distribution for $(\tau_0, \tau_3, \tau_{3S}, \chi_{eff})$ \cdots new parametrization for mass and spin
- We constructed template banks from each type of random samples.

Results of injection test

We checked the accuracy of template bank by preparing 2×10^4 simulated signals and computing the match between a signal and a nearest template point.

common for signals and templates	
waveform model	IMRPhenomD
mass range	$m_1, m_2 \in [10, 100] M_\odot$
spin range	$\chi_1, \chi_2 \in [-0.9, 0.9]$ (aligned spin)
Setting for template bank	
number of random samples	10^4
minimal match	0.97
Setting for simulated signals	
signal parameters	chosen uniformly random

Table 2: Setting of simulated signals and template bank

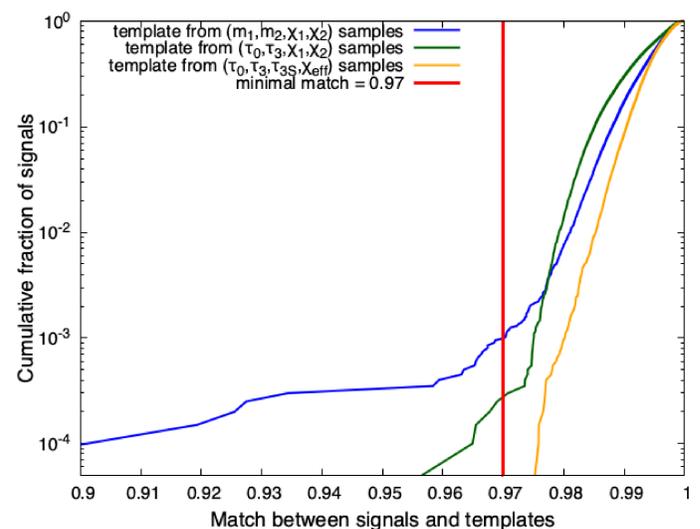


Figure 1: Fraction of signals against to match with templates

We found that the template banks from newly-parametrized samples had better accuracy than the original. We concluded that it's possible to achieve good accuracy even from small number of samples by choosing proper distribution.

Conclusion

- We worked on developing new algorithm to construct template bank for gravitational-wave searches.
- We achieved more accurate results with low computational cost.
- Our new method is also useful in reflecting instrumental updates to analysis rapidly through $S_n(f)$. It may be important if we need to consider the time variation of detector sensitivity.
- We are checking the performance of new algorithm in lower mass regions now.

References

- [1] J. Roulet et al. Template bank for compact binary coalescence searches in gravitational wave data: A general geometric placement algorithm. *Physical Review D*, 99(12), apr 2019.
- [2] H. Tagoshi and T. Tanaka. Use of new coordinates for the template space in a hierarchical search for gravitational waves from inspiraling binaries. *Physical Review D*, 62(8):8, sep 2000.
- [3] D. A. Brown et al. Detecting binary neutron star systems with spin in advanced gravitational-wave detectors. *Physical Review D*, 86(8):084017, oct 2012.
- [4] N. Halko et al. Finding structure with randomness: Stochastic algorithms for constructing approximate matrix decompositions. *SIAM Rev.*, 53(2):217–288, 2011.