Probing Diffuse Dark Matter Haloes with Diffractive Lensing of GW

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Outline

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IV. Summary
I. Motivation

• Dark matter subhalo

According to Cold dark matter (CDM) theory, DM halo has a lot of subhalos.

Test of CDM -> How many? How steep?

Dark matter halo:
- mass $\sim 10^{12} M_\odot$
- size $\sim O(100 \text{kpc})$

Subhalo:
- Mass $< 10^7 M_\odot$
- Size $< 1 \text{kpc}$

According to Cold dark matter (CDM) theory, DM halo has a lot of subhalos.
I. Motivation

Gravitational lensing of Gravitational Wave Chirps
: Sensitive to low mass compact lens
• Intermediate mass black hole (Lai 2018, Jung 2018)
• Dwarf galaxy (Takahashi 2003, Dai 2018)

• Application to Dark matter subhalo?
  • Subhalos are diffuse → Weak gravity
    → Only one signal perturbed by Diffraction
    = Diffractive lensing

We need to understand Diffractive lensing of Gravitational wave.
II. Diffractive lensing: Wave optics

\[(\nabla^2 + w^2)h(w, \mathbf{x}) = 4w^2 U(\mathbf{x}) h(w, \mathbf{x})\]  
Wave propagation

\[\Rightarrow h(w, 0) \sim \left[ \frac{w}{2\pi i d_{\text{eff}}} \int d^2 x' e^{iwT(x')} \right] h_0(w, 0)\]  
Kirchhoff integral

\[F(w) \equiv \frac{w}{2\pi i d_{\text{eff}}} \int d^2 x' e^{iwT(x')}\]  
Lensing amplification factor (F=1 for no lens)

\[h_0(w, 0) : \text{GW without lensing effects}\]
II. Diffractive lensing : Apprx. Solution

\( \tilde{\kappa}(r) \) : (dimless) **Mean surface density** of DM halo within a radius \( r \)

With **weak lensing** assumption, We find that

\[
F(w) \approx 1 + \frac{w}{id_{\text{eff}}} \int_{0}^{\infty} dx x e^{i \frac{wx^2}{d_{\text{eff}}}} \tilde{\kappa}(x)
\]

\[
\Rightarrow F(w) \approx 1 + \bar{\kappa} \left( \frac{r_F e^{i\pi/4}}{\sqrt{2}} \right) \quad r_F \equiv \sqrt{\frac{2d_{\text{eff}}}{w}}
\]

\[
d_{\text{eff}} = \frac{d_l (d_s - d_l)}{d_s} : \text{effective distance between lens and observer}
\]

- \( F(w) \) is equivalent to **DM Halo profile**.

- We can easily find \( F(w) \) **analytically** if mean surface density is given by analytic function.
II. Diffractive lensing: Approx. Solution

Application to more diffuse DM halo profile: Navarro-Frenk-White (NFW) profile

$$\overline{\kappa}(x) = \frac{6\kappa_0}{x^2} \left[ \ln \frac{x}{2} + \mathcal{F}(x) \right]$$

$$\mathcal{F}(x) = \begin{cases} 
\frac{\text{arctanh}\sqrt{1-x^2}}{\sqrt{1-x^2}} & x < 1 \\
1 & x = 1 \\
\frac{\text{arctan}\sqrt{x^2-1}}{\sqrt{x^2-1}} & x > 1 
\end{cases}$$

$$x = r/r_0$$

|F| vs. w

- Solid: Numerical integration
- Dashed: Analytic solution
Due to wave optics effects, a point source has an effective source size (Fresnel length), \( r_F = \sqrt{\frac{2d_{\text{eff}}}{w}} \).

\[
r_F \approx 1.76 \text{pc} \sqrt{\left( \frac{d_{\text{eff}}}{\text{Gpc}} \right) \left( \frac{\text{Hz}}{f} \right)} \sim \text{(sub halo length scale)}
\]

GW chirps from massive Black hole binaries:
low frequency, large source distance, broad spectrum
II. Diffractive lensing : GW spectrum

Diffractive lensing-induced chirping GW spectrum

Lensing by $\kappa(r) \propto r^{-1}$ lens ($M = 10^5 M_\odot, z_l = 0.35$)

![Graph showing strain versus frequency for different BBH masses. Dash: Lensed GW, Solid: Unlensed GW. LISA is marked on the graph.](image-url)
III. Detection prospect: GW detector

- Laser Interferometer space antenna (LISA)
  - Sensitive at 1 mHz

- Big Bang Observer (BBO)
  - Sensitive at 0.1 Hz
III. Detection prospect

\( \dot{N}_L \): Lensing detection per year

- BBO can detect \( 10^{3-4} M_{\odot} \) halo lensing \( O(10) \) per year.
  
  In future, BBO will discriminate CDM and the other DM models.

- LISA and the others are less promising.
  - Lack of **High Signal-to-Noise Ratio**(>1000) BBH sources

\( \dot{N}_L \): Lensing detection per year

BBH Merger rate

Solid : 0.01 \( Gpc^{-3} yr^{-1} \)

Shaded : astrophysical
(Bonetti 2018)
IV. Summary

1. Diffraction effects of GW can be significant due to its large effective source size (Fresnel length $r_F$).

2. We can detect and measure DM subhalo profile by diffractive lensing of GW since $F(w)$ tell us mass distribution at $r_F$ scale.

3. BBO is most promising GW detectors for low mass halo search, which yields $O(10)$ lensing rate for $10^{3-4}M_\odot$ haloes.