Study on an Efficient Template Placement Algorithm for CBC searches in GW data
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Abstract
For searches of gravitational waves from compact binary coalescence, usually matched filtering with template bank is used. For faster and more accurate searches, we introduce a new algorithm for template bank construction based on a previous study[1] using the geometry of parameter space. In our work, we use a technique for matrix computation and consider the distribution of template points. Finally, we confirmed that our new method is more accurate and computationally fast. This algorithm can also respond to instrumental updates rapidly.

Introduction - Template bank
In matched filtering method in GW searches, we evaluate
\[ s(h) = 4\text{Re} \int_0^\infty d\lambda \frac{f^* f}{S_{00}(f)} (\lambda = (m_1, m_2, S_1, \ldots)) \] (1)

Because we don’t know the true parameter value a priori, we need to prepare theoretical waveforms for various parameter values - called “template bank”.

In order to place template points properly, we usually evaluate “match” between two waveforms \( s_i | s_j \). Given a threshold for match (called minimal match (MM)), we compute template points to satisfy the threshold.

Background - Geometry of parameter space
Some studies suggest placement algorithms to use the geometry of parameter space introducing coordinates for \( \lambda \) - called geometric placement.

Such an algorithm becomes simpler by introducing new coordinates for parameters[2]. The phase function \( \psi(f, \lambda) \) of a waveform would be written as linear combination of orthonormal bases \( \{\psi_i(f)\} \) and coefficients \( \{c_i\} \):
\[ \psi(f, \lambda) = \sum c_i(\lambda) \psi_i(f) \] (2)

Because the coordinate for \( \{c_i\} \) becomes Euclidean from orthonormality of bases, the computation of template points on the new coordinate is simple. Previous studies show the efficiency of this method[3].

Even in the case that we don’t know the analytic formula of \( \psi(f, \lambda) \), we can identify \( \{c_i\} \) and \( \{\psi_i\} \) using singular value decomposition (SVD) as follows[4]:
1. Generate a lot of random samples for parameters, \( \{\lambda\} \).
2. Compute discrete phase matrix, \( \Psi_{ij} = \psi(f_i, \lambda_j) \).
3. Execute SVD for the phase matrix. \( \implies \) search of orthonormal coordinates and bases
Then we can construct template bank on the new Euclid coordinate for \( \{c_i\} \).

Improvement - Less computation and More accuracy
1. Fast computation of SVD
In previous method, we needed to execute SVD for a huge size matrix. It dominates computational time in template construction.

To reduce computational cost in SVD, we introduced an approximation technique for low-rank matrix which is proposed in computer science originally[4]. In this method, we approximate a matrix \( A \in \mathbb{R}^{M \times N} \) by \( A' \in \mathbb{R}^{M \times L} \) (\( R \ll M \)) extracting principal components from original matrix.

Performance test
We tested this method for the matrix \( \Psi_{01} \) which has \( 10000 \times 16384 \) components. In my laptop, computational time of SVD was

\[ \begin{array}{|c|c|}
\hline
\text{w/o approximation} & 10 \text{hr.} \\
\text{w/ approximation} & 6 \text{min.} \\
\hline
\end{array} \]

Table 1: Performance of matrix approximation
with relative error \( < 10^{-20} \). Approximate SVD needed only \( \mathcal{O}(10^3) \) principal components in the original matrix. We confirmed that matrix approximation works effectively to reduce computational cost in template construction.

2. Flatten the distribution of parameter points
In previous method, we needed to use a lot of random samples, and the accuracy of template bank is affected by the distribution of random samples.

To improve accuracy, we tried to flatten the distribution of random samples in the parameter space NOT for \( \lambda \) BUT for \( e \). We tested three types of distribution of random samples for the mass and spin of binary objects,

1. Uniform distribution for \( (m_1, m_2, \chi_1, \chi_2) \)
2. Uniform distribution for \( (\tau_0, \tau_1, \chi_1, \chi_2) \) - new parametrization for mass
3. Uniform distribution for \( (\tau_0, \tau_1, \tau_2, \chi_1, \chi_2) \) - new parametrization for mass and spin

We constructed template banks from each type of random samples.

Results of injection test
We checked the accuracy of template bank by preparing \( 2 \times 10^3 \) simulated signals and computing the match between a signal and a nearest template point.

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<th>common for signals and templates</th>
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<tr>
<td>waveform model</td>
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<td>mass range</td>
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<td>spin range</td>
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<th>Setting for template bank</th>
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<tr>
<td>number of random samples</td>
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<td>minimal match</td>
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<th>Setting for simulated signals</th>
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<td>signal parameters</td>
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![Figure 1: Fraction of signals against to match with templates](image)

We found that the template banks from newly-parametrized samples had better accuracy than the original. We concluded that it’s possible to achieve good accuracy even from small number of samples by choosing proper distribution.

Conclusion
• We worked on developing new algorithm to construct template bank for gravitational-wave searches.
• We achieved more accurate results with low computational cost.
• Our new method is also useful in reflecting instrumental updates to analysis rapidly through \( S_{00}(f) \). It may be important if we need to consider the time variation of detector sensitivity.
• We are checking the performance of new algorithm in lower mass regions now.

References