Mass-spin Re-Parameterization for Rapid PE of Inspiral Gravitational-Wave Signals

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Objectives
The objective of our research is to reduce the time for Bayesian estimation of parameters of inspiral gravitational-wave, keeping the estimation results unchanged. For this, we introduce an alternative set of mass-spin parameters for Markov chain Monte Carlo (MCMC) sampling.

Methods
The restricted post-Newtonian waveform in 1.5PN can be represented as

\[ h(f) = \mathcal{A} \left( \frac{f}{f_{\text{ref}}} \right)^{\frac{5}{2}} e^{-i\psi(f)} \]  

with the phase function \( \psi(f) = \psi^i \left( \frac{f}{f_{\text{ref}}} \right)^{-\frac{1}{2}} + \psi^j \left( \frac{f}{f_{\text{ref}}} \right)^{-\frac{3}{2}} + \psi^k \left( \frac{f}{f_{\text{ref}}} \right)^{-\frac{5}{2}} \).

\[ \psi^i \text{ is a known function of physical parameters, such as masses and spins. The new parameters are } \mu_{ij} = \sum_k U_{ij} \mu^k, \]  

where \( U_{ij} \) is an orthogonal matrix that diagonalizes the Fisher matrix for \( \psi^i, \Gamma_{ij} \equiv \partial h/\partial \psi^i, \partial h/\partial \psi^j \).

Advantages
Using \( \mu \) as sampling parameters makes the posterior distribution to be simple in the parameter space. Assuming Gaussian noise, the log-likelihood at \( \mu = \mu_0 + \Delta \mu \) can be approximated as

\[ p(\mu | d) \propto p(\mu) \prod e^{-\frac{1}{2} \sum \lambda_{ij} \Delta \mu_i \Delta \mu_j}, \]

where \( p(\mu) \) is the prior distribution. We can generate posterior samples efficiently in \( \mu \) space, and get the posterior distribution for physical parameters easily by converting the generated samples.

Implementation

<table>
<thead>
<tr>
<th>Case Spin prior</th>
<th>( m_1[M_0] )</th>
<th>( m_2[M_0] )</th>
<th>( \chi_1 )</th>
<th>( \chi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 Narrow</td>
<td>1.8839</td>
<td>1.8839</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>#2 Semi-broad</td>
<td>1.8839</td>
<td>1.8839</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>#3 Broad</td>
<td>2.2588</td>
<td>1.5811</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1: Test cases.

We compare the parameter estimation with and without our re-parameterization using BILBY [1] and PTMCMCSampler [2]. The test is done in 3 cases of Table 1, which are different in spin prior range.

Without and with re-parameterization, the sampling parameters are \( \{ M, q, \chi_1, \chi_2, \phi_1, \phi_2 \} \) and \( \{ \mu_1, \mu_2, q, \chi_1, \chi_2, \phi_1, \phi_2 \} \) respectively.

To evaluate the sampling efficiency of MCMC, the maximum integrated auto-correlation time (IAT) is used [3]. Each iteration of MCMC generates a posterior sample, but the samples are auto-correlated. Maximum IAT shows the required number of iterations equivalent to one independent sample.

Results

To evaluate the sampling efficiency of MCMC, the maximum integrated auto-correlation time (IAT) is used [3]. Each iteration of MCMC generates a posterior sample, but the samples are auto-correlated. The maximum IAT shows the required number of iterations equivalent to one independent sample.

Table 2 shows the estimated maximum IATs. In case #2, the maximum IAT is reduced by 1/29, and it implies the reduction of estimation time by 1/29 for the identical estimation precision.

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Without and with re-parameterization, the sampling parameters are \( \{ M, q, \chi_1, \chi_2, \phi_1, \phi_2 \} \) and \( \{ \mu_1, \mu_2, q, \chi_1, \chi_2, \phi_1, \phi_2 \} \) respectively.

We assume a single detector and 128/s/2048Hz data. Injected and searching waveform is IMRPhenomD approximant. Geometrical parameters are chosen arbitrarily to make SNR to be around 10.

Conclusion

In this work, we have introduced a new set of mass-spin parameters for compact binary inspiral waveform, which makes the posterior distribution simple. In all injection tests, the new parameters improved the efficiency of the sampling process. Especially the improvement was remarkable when posterior distribution had complicated shape in the usual mass-spin parameter space, like broader spin prior range cases.

References


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Table 2: The estimated maximum IAT values estimated from the samples.